Stabilizing weighted complex networks

2007 J. Phys. A: Math. Theor. 40 14369

(http://iopscience.iop.org/1751-8121/40/48/004)

IOP Publishing — Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. **40** (2007) 14369–14382 doi:10.1088/1751-8113/40/48/004

Stabilizing weighted complex networks

Linying Xiang, Zengqiang Chen, Zhongxin Liu, Fei Chen and Zhuzhi Yuan

Department of Automation, Nankai University, Tianjin 300071, People's Republic of China

E-mail: xlyzhjl1980@gmail.com and chenzq@nankai.edu.cn

Received 13 July 2007, in final form 23 October 2007
Published 14 November 2007
Online at stacks.iop.org/JPhysA/40/14369

Abstract
Real networks often consist of local units which interact with each other via asymmetric and heterogeneous connections. In this paper, the V-stability problem is investigated for a class of asymmetric weighted coupled networks with nonidentical node dynamics, which includes the unweighted network as a special case. Pinning control is suggested to stabilize such a coupled network. The complicated stabilization problem is reduced to measuring the semi-negative property of the characteristic matrix which embodies not only the network topology, but also the node self-dynamics and the control gains. It is found that network stabilizability depends critically on the second largest eigenvalue of the characteristic matrix. The smaller the second largest eigenvalue is, the more the network is pinning controllable. Numerical simulations of two representative networks composed of non-chaotic systems and chaotic systems, respectively, are shown for illustration and verification.

PACS numbers: 05.45.Pq, 05.45.−a, 89.75.−K

(Some figures in this article are in colour only in the electronic version)

1. Introduction

We live in a world of networks. In fact any large-scale and complicated system in nature and societies can be modeled by a complex network, where vertices are the elements of the system and edges represent the interactions between them. Examples include the WWW, the Internet, metabolic networks, neural networks, food webs, electrical power grids, social networks and many others. Recent technological advances have spurred a broad interest in the study of complex networks, including modeling [1–6], synchronization [7–32], control [33–41], epidemic dynamics [42, 43], etc.

Greatly differing from general dynamical systems, the collective behavior of a complex network is highly determined not only by the self-dynamics governing the isolated nodes, but also by the overall topology of the network, between which the interplay gives rise to a host of interesting effects. In particular, there are attempts to control the aggregate dynamics of a complex network and guide it to a desired state, where pinning control was proposed as a viable control scheme with minimal number of controllers placing at the nodes [33–41]. A basic assumption characterizing most of the previous work on this issue [34–40] is that all the network dynamical nodes are identical, coupling symmetrically with the same coupling strength. However, in many circumstances this simplification does not match the peculiarities of real networks with satisfaction. For instance, some phenomena such as the diversity of the predator–prey interactions in food webs [44, 45], different capabilities of transmitting electric signals in neural networks [46, 47], unequal traffic on the Internet [48] or of the passengers in airline networks [49, 50] explain the existence of weighted wirings [51]. The distribution of connection weights in realistic networks is often highly heterogeneous. A first weighting approach to enhancing network synchronization was proposed in [52], where the effects of variable weights depend essentially on the node degrees. Such a weighting procedure is convenient in that by varying the weight parameter, it is possible to tune the coupling strength among the network nodes according to their degrees. A new notion of Lyapunov V-stability for a class of symmetric network with different nodes was first introduced in [41], where the impact of the node self-dynamics on the network stability is shifted to the corresponding passivity degree, a scalar parameter indicating the extent of the effort needed to stabilize the node. In [34], a different strategy based on pinning control was addressed by introducing an extra virtual node (or a reference node) into the original network. Therein, the arguments of the master stability function approach can be immediately applied to the appropriately extended network.

In this paper, the V-stability problem is further revisited for a class of asymmetric weighted network with different node dynamics, which includes unweighted network as a special case. In particular, the nodes in the network are not assumed to be identical. Pinning control is applied to stabilize the network. It is found that the stabilizability depends critically on the second largest eigenvalue of the characteristic matrix. The smaller the second largest eigenvalue is, the more the network is pinning controllable. Note that differing from the Laplacian or adjacent matrix characterizing the network topology in most previous eigenvalue-based studies [7, 9, 11, 13, 21, 25], the characteristic matrix here encompasses not only the network topology in terms of the connections and the weights over them, but also the node self-dynamics and the control gains over them.

The remainder of the paper is outlined as follows. A new model of weighted network composing of nonidentical nodes is presented and some preliminaries are introduced in section 2. In section 3, the V-stability conditions are obtained, which guarantee that the whole network can be pinned to its equilibrium by placing pinning controllers at a much small fraction of nodes. Some numerical simulations for verifying the theoretical results are given in section 4. Finally, section 5 concludes the investigation.

2. Model description and preliminaries

Consider a network consisting of N different nodes interacting through a diffusive-type coupling, where each node is an m-dimensional dynamical system, described by

$$\dot{x}_i = f_i(x_i) - \frac{a}{k_i^{\beta}} \sum_{j=1, j\neq i}^{N} l_{ij}\Gamma(x_j - x_i), \qquad i = 1, 2, \dots, N, \tag{1}$$

where $x_i \in R^m$ represents the state vector of the ith node, and the function $f_i(\cdot)$, governing the self-dynamics of node i is capable of producing various rich dynamical behaviors, including periodic orbits and chaotic states. The parameter a is positive ruling the overall coupling strength; k_i is the out-degree of node i and β is a tunable weight parameter. Also, $\Gamma \in R^{m\times m}$ is a constant matrix linking coupled variables, while the real matrix $L = (l_{ij})_{i,j=1}^{N}$ is the usual (symmetric) Laplacian matrix [10, 39, 52].

Recall that the parameter $\beta = 0$ recovers that the network is unweighted and undirected, and the condition $\beta \neq 0$ corresponds to a network with not only weighted but also directed configuration. Although it is unnecessary to have a unique globally connected cluster to control a network, while in each isolated cluster, at least one controller is present [35]. For the sake of simplicity, this paper only focuses on the case that network (1) is diffusively connected without isolated clusters. It then follows from the well-known Perron–Frobenius theorem that the matrix L is irreducible.

For notational convenience, the original network (1) can be rewritten as

$$\dot{x}_i = f_i(x_i) + a\sum_{j=1}^{N} b_{ij}\Gamma x_j, \qquad i = 1, 2, \ldots, N, \tag{2}$$

where $B = (b_{ij})_{i,j=1}^{N}$ is a coupling matrix of the directed weighted network, defined by

$$b_{ij} = -l_{ij}\big/k_i^{\beta}. \tag{3}$$

Note that the asymmetric matrix B is irreducible and negative semi-define since the network considered here is diffusively connected. It follows that B has real eigenvalues and can be diagonalizable [10, 39, 52].

Assume that all the network nodes have a common equilibrium $\bar{x} \in R^m$, satisfying $f_i(\bar{x}) = 0$ for $i = 1, 2, \ldots, N$. The task here is to stabilize network (2) onto the reference state $x_1 = \cdots = x_N = \bar{x}$.

Before proceeding with the analytic treatment, the following assumption and lemma are made throughout the paper.

Assumption 1 [41]**.** There exists a continuously differentiable Lyapunov function $V(x)$ satisfying $V(\bar{x}) = 0$ with $\bar{x} \in D$, such that for each node function $f_i(x_i)$, there is a scalar θ_i called passivity degree holding

$$\begin{aligned}&\frac{\partial V(x_i)}{\partial x_i}(f_i(x_i) - \theta_i\Gamma(\bar{x} - x_i)) < 0,\\ &\qquad \forall x_i \in D_i, \qquad x_i \neq \bar{x}, \qquad i = 1, 2, \ldots, N,\end{aligned} \tag{4}$$

where $D_i = \{x_i : \|x_i - \bar{x}_i\| < \alpha\}$,$\alpha > 0, D = \bigcup_{i=1}^{N} D_i$.

Lemma 1 [41]**.** Define $\bar{D} = D_1 \times D_2 \times \cdots \times D_N \subseteq R^{mN}$. Consider the following Lyapunov function for the network (2):

$$V_N(X) = \sum_{i=1}^{N} V(x_i), \qquad X = \left(x_1^T, x_2^T, \ldots, x_N^T\right)^T. \tag{5}$$

If there exists a function $G(X) \leqslant 0$ such that $\dot{V}_N(X) < G(X)$ for all $X \in \bar{D}\backslash\{0\}$, then network (2) is locally asymptotically stable about its equilibrium point. Moreover, the region of attraction is given by

$$\Omega = \{X : V_N(X) < r\}, \qquad r = \inf_{X\in\bar{D}} V_N(X). \tag{6}$$

In the case of $\bar{D} = R^{mN}$, the above stability becomes global.

3. Main results

The control objective can be achieved via pinning control. Without loss of generality, let the first l ($l \ll N$) nodes be controlled with simple pinning controllers of the form $u_i = -ad_i\Gamma(x_i - \bar{x})$, $i = 1, 2, \ldots, l$, where the feedback control gain $d_i > 0$ acting on node i needs to be designed. Thus, the self-dynamics of the pinned nodes becomes $\dot{x}_i = f_i(x_i) - ad_i\Gamma(x_i - \bar{x})$, $i = 1, 2, \ldots, l$. It is worth noting here that the networks are divided into two different layers of dynamical nodes: the un-pinned nodes and the pinned ones. In particular, the latter play the role of network leaders leading the entire network toward a given desired stationary state. The direct control action is propagated to the rest of the network through the coupling among the nodes.

Naturally, the controlled network can be described by the following set of equations:

$$\dot{x}_i = f_i(x_i) - ad_i\Gamma(x_i - \bar{x}) + a\sum_{j=1}^{N} b_{ij}\Gamma x_j, \qquad i = 1, 2, \ldots, N, \tag{7}$$

with $d_i > 0$ for $i = 1, 2, \ldots, l$ and $d_i = 0$ otherwise.

Based on the concept of extended network introduced in [34], let $y_i = x_i$ for $i = 1, 2, \ldots, N$ and $y_{N+1} = \bar{y} = \bar{x}$ corresponding to $\theta_{N+1} = 0$. Then one has

$$\dot{y}_i = f_i(y_i) + a\sum_{j=1}^{N+1} M_{ij}\Gamma y_j, \qquad i = 1, 2, \ldots, N+1, \tag{8}$$

where

$$M = \begin{pmatrix} b_{11} - d_1 & b_{12} & \cdots & b_{1N} & d_1 \\ b_{21} & b_{22} - d_2 & \cdots & b_{2N} & d_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NN} - d_N & d_N \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}. \tag{9}$$

Note that the $(N+1)$-dimensional square matrix M is an asymmetric zero row-sum matrix, with nonpositive values along the main diagonal. From the above it follows that B has real eigenvalues and can be diagonalizable, which ensures the matrix M to be diagonalizable. By graph theory and linear algebra, M is a negative semi-define matrix with a single zero eigenvalue [34].

Remark 1. The desired common state $\bar{x}$ is given by the state of an extra virtual node, indexed by $N+1$, which is added to the original network. The purpose of introducing the virtual node is to ensure some existing approaches can be immediately applied to equation (8). In what follows, the stability problem of the extended network (8) about $\bar{Y} = (\bar{y}^T, \bar{y}^T, \ldots, \bar{y}^T)^T \in R^{m(N+1)}$ is investigated in detail.

Theorem 1. Assume that there exist functions $V(y_i) = \frac{1}{2}y_i^T Q y_i$, $i = 1, 2, \ldots, N+1$, with Q being a symmetric and positive definite matrix, satisfying assumption 1 with passivity degree value θ_i, such that the following inequality holds:

$$Q\Gamma + \Gamma^T Q \geqslant 0. \tag{10}$$

Then, the controlled network (7) is V-stable if the following inequality is satisfied:

$$-\Theta + \tfrac{1}{2}a(M + M^T) \leqslant 0, \tag{11}$$

where $\Theta = \mathrm{diag}(\theta_1, \theta_2, \ldots, \theta_{N+1}) \in R^{(N+1)\times(N+1)}$. Moreover, if $\bar{D} = R^{m(N+1)}$, the above stability is global.

Proof. Without loss of generality, assume $\bar{Y} = 0$ and consider the following Lyapunov function for the extended network (8):

$$V_{N+1}(Y) = \sum_{i=1}^{N+1} V(y_i), \qquad Y = \left(y_1^T, y_2^T, \ldots, y_{N+1}^T\right)^T. \tag{12}$$

Its time derivative along trajectory Y is given by

$$\begin{aligned}\dot{V}_{N+1}(Y) &= \sum_{i=1}^{N+1} \frac{\partial V(y_i)}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}t} \\ &= \sum_{i=1}^{N+1} \frac{\partial V(y_i)}{\partial y_i} f_i(y_i) + a\sum_{i=1}^{N+1} \frac{\partial V(y_i)}{\partial y_i}\sum_{j=1}^{N+1} M_{ij}\Gamma y_j.\end{aligned} \tag{13}$$

It is easy to say that $V_{N+1}(\bar{Y}) = 0$ and $\dot{V}_{N+1}(\bar{Y}) = 0$. Also, assumption 1 implies that, for $Y \neq \bar{Y}$, one has

$$\dot{V}_{N+1}(Y) < G(Y), \tag{14}$$

where

$$G(Y) = \sum_{i=1}^{N+1} \frac{\partial V(y_i)}{\partial y_i}\left(-\theta_i\Gamma y_i + a\sum_{j=1}^{N+1} M_{ij}\Gamma y_j\right). \tag{15}$$

Since $\frac{\partial V(y)}{\partial y} = y^T Q$, the function $G(Y)$ defined in (15) can be written as

$$\begin{aligned}G(Y) &= \sum_{i=1}^{N+1} y_i^T Q\left(-\theta_i\Gamma y_i + a\sum_{j=1}^{N+1} M_{ij}\Gamma y_j\right) \\ &= Y^T(-\Theta + aM)\otimes(Q\Gamma)Y.\end{aligned}$$

From (10) and (11), the matrix $(-\Theta + aM)\otimes(Q\Gamma)$ is negative semi-define. And from lemma 1, the controlled network (7) is V-stable. The proof is finished. □

Remark 2. In the following, the matrix $C = -\Theta + \frac{1}{2}a(M + M^T)$ is called *characteristic matrix*, which differs from the Laplacian or adjacent matrix characterizing only the network topology in most previous eigenvalue-based studies [7, 9, 11, 13, 21, 25]. The characteristic matrix here contains not only the network topology in terms of the connections and the weights over them, but also the node self-dynamics and the control gains over them. Under the proposed V-stability scheme, the network stability problem is then shifted to measuring the semi-negative property of the characteristic matrix. On the other hand, from assumption 1, the passivity degree value is determined critically by the selection of the common Lyapunov function. This is why the stability obtained here is called *V-stability*.

Remark 3. From condition (11), if all the node passivity degree values $\theta_i > 0$ $(i = 1, 2, \ldots, N)$, the network itself is always stable without external control. It can also be seen that the larger the passivity degree value θ_i, the more likely the condition (11) is held.

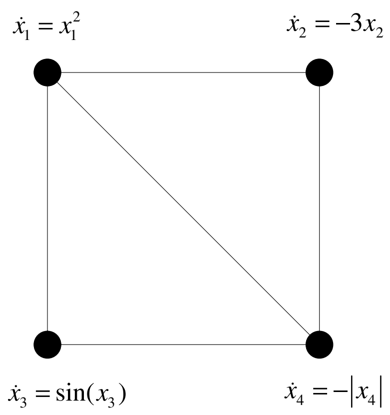

Figure 1. The topological structure and self-dynamics of network (16).

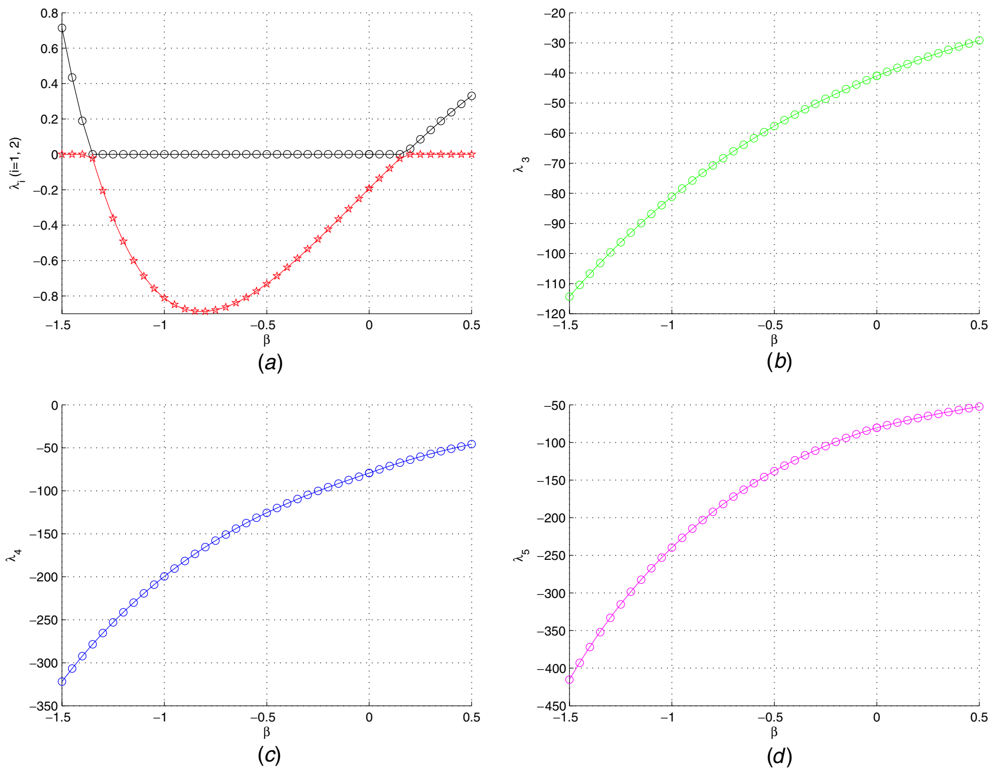

Figure 2. The relation between the eigenvalues of the characteristic matrix and the weight parameter $\beta : \gamma \approx 0$ and $a = 20$. In (a), black circles are used for λ_1 and red stars are used for λ_2.

Remark 4. Although theorem 1 is given for such a diagonalizable weighted network, it can be extended also to the case of non-diagonalizable networks which embeds at least an oriented spanning tree [31, 32].

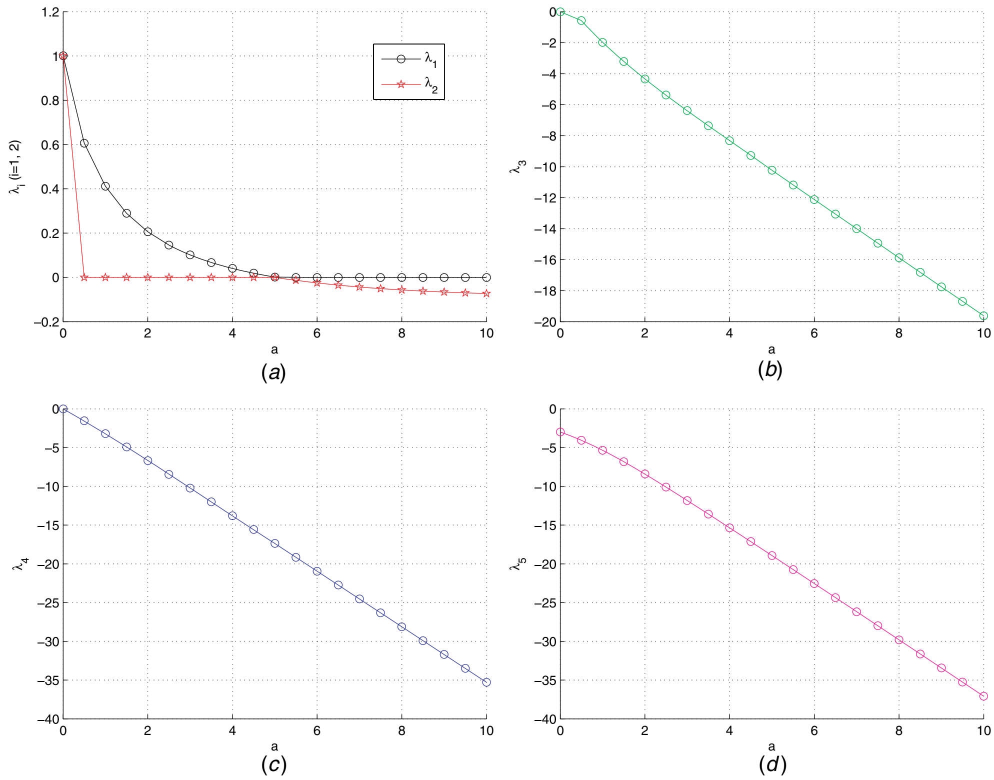

Figure 3. The relation between the eigenvalues of the characteristic matrix and the overall coupling strength $a : \gamma \approx 0$ and $\beta = 0.1$.

4. Simulation study

4.1. Non-chaotic networks

To validate the theoretical results, consider a simple weighted network with four different nodes, as shown in figure 1. The node local dynamics are given by [41]:

$$\dot{x}_1 = x_1^2, \qquad \dot{x}_2 = -3x_2, \qquad \dot{x}_3 = \sin(x_3), \qquad \dot{x}_4 = -|x_4|. \tag{16}$$

Obviously, $\bar{x} = 0$ is the common equilibrium point. Note that the function in the fourth node is not differential at the equilibrium, so any linearization method is not preferable. However, the limitation of linearization is avoided by introducing an important scale, θ, as a measure of the effect of the node self-dynamics on the network stability.

Take the common Lyapunov function $V(X) = X^T X$, then the corresponding passivity degrees of the four nodes are $\theta_1 = -\gamma, \theta_2 = 2.999, \theta_3 = -1.001$ and $\theta_4 = -1.001$ with $\gamma > 0$. The corresponding feasible domains are $D_1 = \{x_1 : |x_1| < \gamma\}$ and $D_2 = D_3 = D_4 = R$. The region of attraction is then given by

$$\Omega = \left\{(x_1, x_2, x_3, x_4) : x_1^2 + x_2^2 + x_3^2 + x_4^2 < \gamma^2\right\}. \tag{17}$$

Thus, the characteristic matrix of the uncontrolled network (16) can be rewritten as

$$C_u = -\Theta + \tfrac{1}{2}a(M + M^T), \tag{18}$$

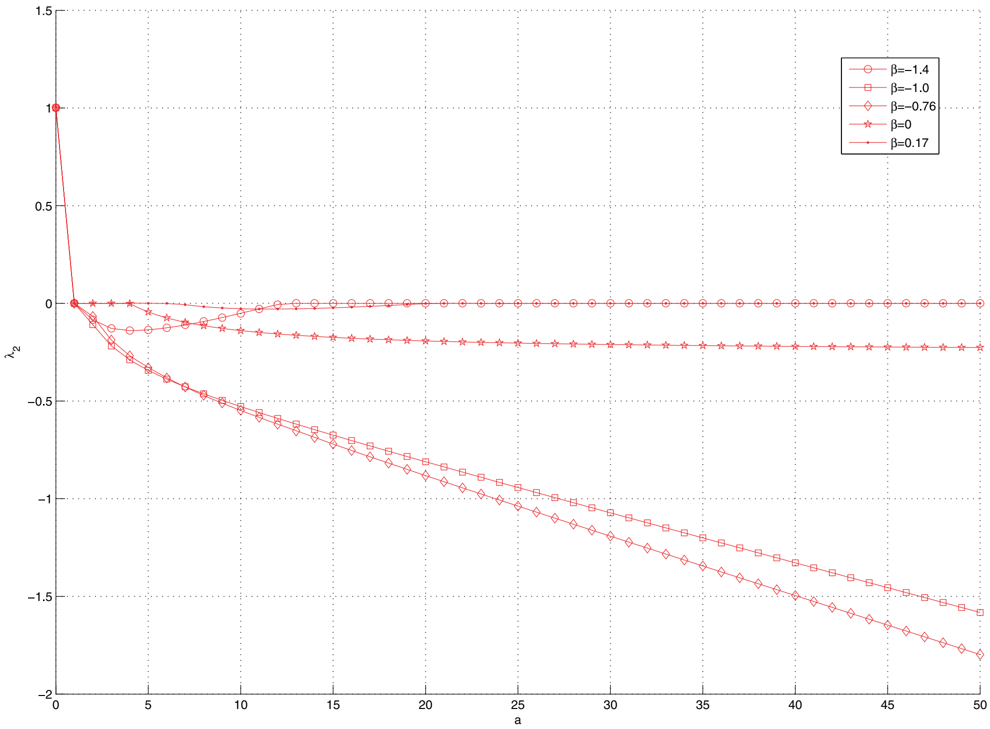

Figure 4. The relation between the second largest eigenvalue of the characteristic matrix and the overall coupling strength a. The legend is as follows: $\beta = -1.4$ (circles); $\beta = -1$ (squares); $\beta = -0.76$ (diamonds); $\beta = 0$ (stars); $\beta = 0.17$ (dots).

where

$$\Theta = \begin{pmatrix} -\gamma & 0 & 0 & 0 & 0 \\ 0 & 2.999 & 0 & 0 & 0 \\ 0 & 0 & -1.001 & 0 & 0 \\ 0 & 0 & 0 & -1.001 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad M = \begin{pmatrix} -\frac{3}{3^\beta} & \frac{1}{3^\beta} & \frac{1}{3^\beta} & \frac{1}{3^\beta} & 0 \\ \frac{1}{2^\beta} & -\frac{2}{2^\beta} & 0 & \frac{1}{2^\beta} & 0 \\ \frac{1}{2^\beta} & 0 & -\frac{2}{2^\beta} & \frac{1}{2^\beta} & 0 \\ \frac{1}{3^\beta} & \frac{1}{3^\beta} & \frac{1}{3^\beta} & -\frac{3}{3^\beta} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

with the arranged eigenvalues denoted by $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \lambda_4 \geqslant \lambda_5$.

In the following, the effects of the four important parameters γ, a, β and d on the eigenvalues of the characteristic matrix are investigated, respectively.

Firstly, consider the worst case, i.e., take the limiting value $\gamma = 0$. Fix $a = 20$ and calculate the eigenvalues from $\beta = -1.5$ to 0.5, with step size 0.01, so as to obtain the numerical results shown in figure 2. As β increases, first a decrease and then an increase of the second largest eigenvalue is observed. In particular, $-1.34 \leqslant \beta \leqslant 0.17$ is required to stabilize the network (16). Furthermore, a pronounced minimum for the second largest eigenvalue at $\beta = -0.76$ is observed in figure 2(*a*).

Set $\beta = 0.1$ in the worst case of $\gamma \approx 0$. Similarly, the relationship between the overall coupling strength a and the eigenvalues is illustrated in figure 3. As β is fixed and a increases, all the eigenvalues decrease. Eventually, all the eigenvalues become negative (except the largest one is 0) when $a > 5$, at which the network will evolve to be locally stable from

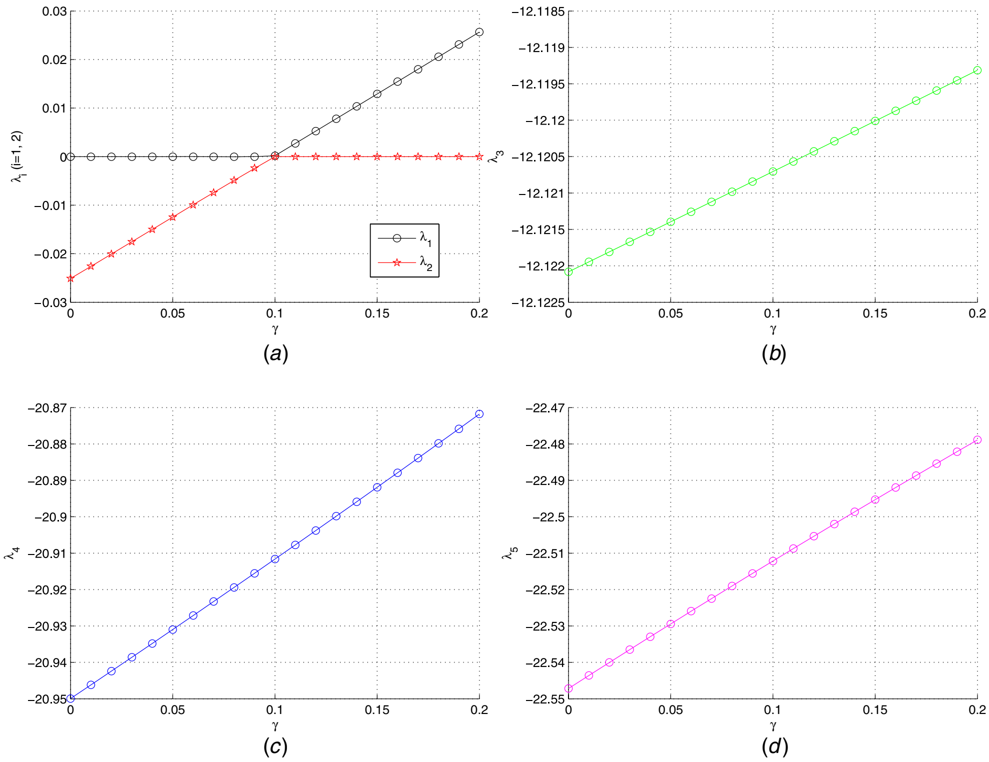

Figure 5. The relation between the eigenvalues of the characteristic matrix and γ : $\beta = 0.1$ and $a = 6$.

unstable. Naturally, in such a case, the stronger the overall coupling strength a, the easier the stabilization.

In figure 4, the second largest eigenvalue has been plotted versus the coupling strength a corresponding to different β. It is intuitional that the larger the a, the easier the stabilization. However, by increasing a, more complex behavior emerges: for some β, the second largest eigenvalue does not always decrease as a grows. A supporting explanation is that positive (negative) values of β indicate that the strengths of the couplings acting on each node decrease (increase) with its degree. This indicates the existence of optimal ranges of values of β and a in terms of the network stabilizability.

Fix $\beta = 0.1$ (corresponding to an asymmetric network), $a = 6$ and plot the changes of the eigenvalues versus γ as shown in figure 5. Similar to figure 2, $0 < \gamma < 0.1$ is needed to stabilize. It then follows from (17) that the V-stability of network (16) can be ensured but only in a small region of attraction: $\Omega = \{X : \|X\| < 0.1\}$. To enlarge this region of attraction and obtain high control efficiency, pinning control is the best choice.

Now, in the case often considered in the literature of all control gains being the same, i.e., $d_1 = d_2 = \cdots = d_l = d$. In view of $\gamma = 10.0001$, $\beta = 0.1$ and $a = 6$, design the control law as $u_1 = -a\,dx_1$. The relation between the eigenvalues of the characteristic matrix and the control gain d is given in figure 6. It is clear that the eigenvalues do not increase as d increases and the aim state can be stabilized if $d \geqslant 1.7$. Figure 7 shows the evolution process of the network state. The network (16) diverges from the initial state $X(0) = (0, 0, 10, 0)^T$ without

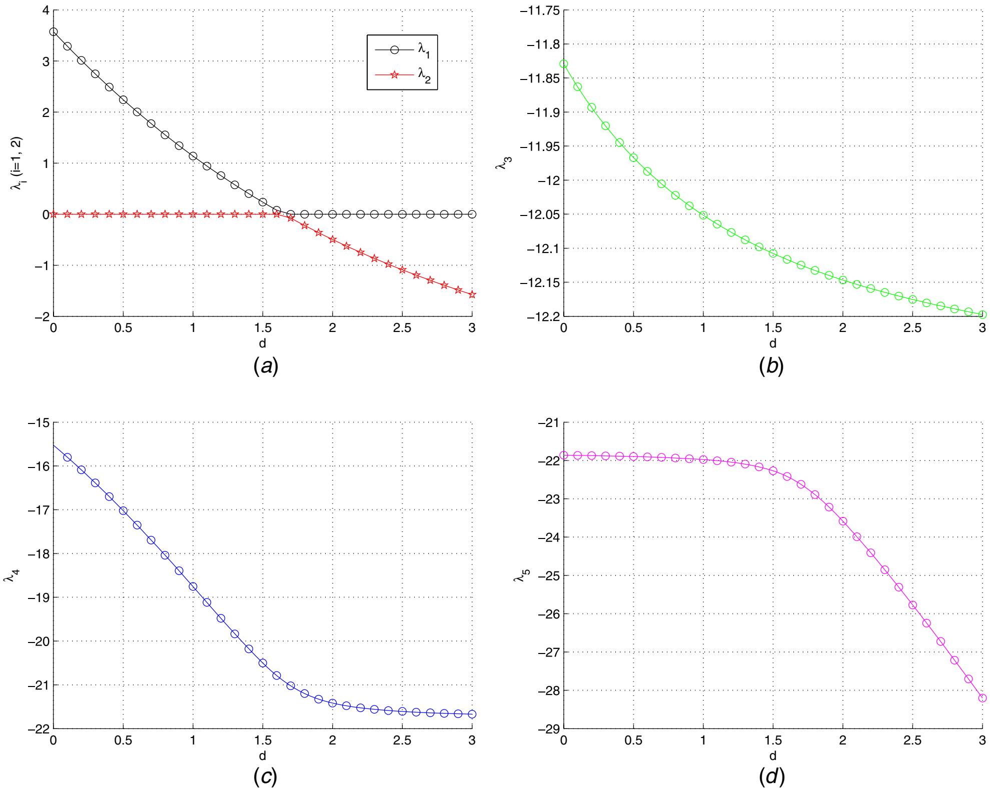

Figure 6. The relation between the eigenvalues of the characteristic matrix and the control gain d : $\gamma = 10.0001$, $\beta = 0.1$ and $a = 6$.

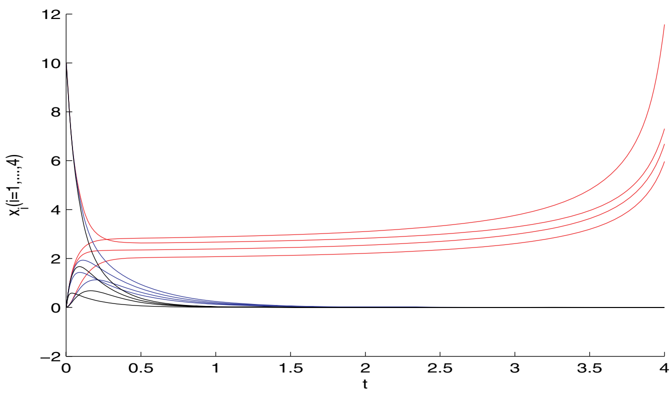

Figure 7. The evolution of network states with initial state $X(0) = (0, 0, 10, 0)^T$: the red lines denote the state without control, the blue and black lines represent the states with the control gain $d = 2$ and $d = 10$, respectively.

control but converges quickly to the origin after control. It is concluded that increasing the feedback gain can considerably enhance the controlling efficiency. That is to say, the larger the control gain is, the better the network is stabilized.

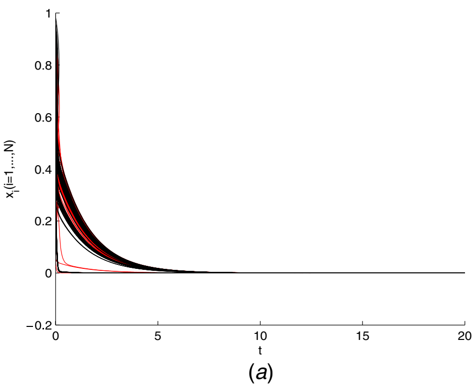

Figure 8. Specifically pinning the 'biggest' Lorenz node of out-degree 20 in a 50-node weighted random network: parts (*a*)–(*d*) are stabilizing phase trajectories with different coupling strengths. (*a*) $a = 100$, $d = 100$, (*b*) $a = 110$, $d = 100$. (The red lines denote the states of Lorenz nodes and black lines represent Chen nodes. These notations are valid for all the following figures.)

Motivated by the above analysis, for given node dynamics of a linearly weighted coupled network, three key factors influencing the stabilizability are the overall coupling strength a, the weight parameter β and the feedback control gain d. The larger the d, the more stabilizable the network. Also there exist optimal ranges of values of β and a such that the second largest eigenvalue of the characteristic matrix, λ_2, is minimal. The λ_2 is directly determined by these three important parameters. On the other hand, for a given topology network, the passivity degree θ and the control gain d are the significant factors influencing the stabilizability. The larger the θ and d, the easier the stabilization. The λ_2 is directly dominated by these two important parameters. In sum, the eigenvalue λ_2 determines the stabilizability of the network. No matter what the network is, the smaller the λ_2 is, the more the network is pinning controllable.

4.2. Chaotic networks

In this subsection, chaotic network is simulated to further illustrate the proposed V-stability scheme.

Consider a 50-node weighted random network consisting of two kinds of node self-dynamics: the Lorenz system $L(x)$ [53] and the Chen system $C(x)$ [54], where

$$L(x): \quad \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \alpha_L(x_2 - x_1) \\ \gamma_L x_1 - x_2 - x_1 x_3 \\ x_1 x_2 - \beta_L x_3 \end{pmatrix}, \tag{19}$$

$$C(x): \quad \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \alpha_C(x_2 - x_1) \\ (\gamma_C - \alpha_C)x_1 + \gamma_C x_2 - x_1 x_3 \\ x_1 x_2 - \beta_C x_3 \end{pmatrix}. \tag{20}$$

When $\alpha_L = 10$, $\beta_L = 8/3$ and $\gamma_L = 28$, the Lorenz system is in a chaotic state, so is the Chen system with $\alpha_C = 35$, $\beta_C = 3$ and $\gamma_C = 28$. The number of Lorenz-type nodes is the same as the Chen-type nodes. Consider $\beta = 0.1$, $\Gamma = \mathrm{diag}(0, 1, 1)$ and $V(x) = x^T P x$ with $P = \mathrm{diag}(0.1, 1, 1)$. From assumption 1, one has $\theta_L = -29$ and $\theta_C = -2$ regarding

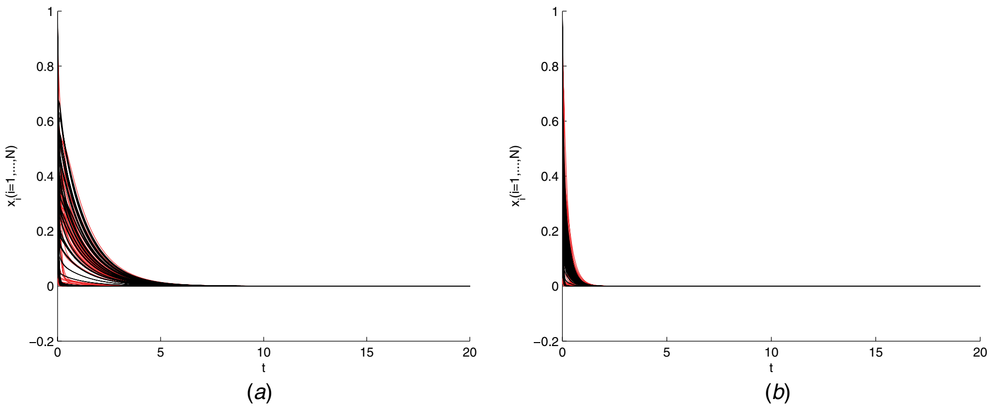

Figure 9. Specifically pinning five 'biggest' Lorenz nodes of out-degrees 20, 19, 17, 11 and 10 in the same 50-node network: parts (*a*) and (*b*) are stabilizing phase trajectories with different coupling strengths. (*a*) $a = 30$, $d = 100$, (*b*) $a = 40$, $d = 100$.

the common equilibrium $\bar{x} = [0, 0, 0]^T$. The initial values of the network are in the uniform distribution on the interval (0, 1).

Figure 8 shows the stabilization process with the 'biggest' Lorenz node pinned, which has out-degree 20. According to theorem 1, when the controller with feedback gain $d = 100$ is used, the corresponding stable threshold of the overall coupling strength a is 64.

Figure 9 shows the same stabilization process with five 'biggest' Lorenz nodes pinned, which have out-degrees 20, 19, 17, 11 and 10, respectively, leading to a smaller threshold for the overall coupling strength a, 20.4, with the same control gain $d = 100$.

It can be observed that increasing the overall coupling strength a and the pinning fraction δ will accelerate the convergence of the network stabilization.

5. Conclusions

In this paper, a new model of weighted coupled network with different nodes has been introduced and studied, and the V-stability problem of such a network has been investigated by pinning a tiny portion of nodes with negative feedback controllers. Under the V-stability framework, the network stability is converted to determining the semi-negative definiteness of one simple characteristic matrix which characterizes the node dynamics and the topology of the extended network. Furthermore, the impact of the node self-dynamics on the network stability is shifted to the corresponding passivity degree, so the node function is only bounded without the requirement of differentiability. It is also shown that the second largest eigenvalue of the characteristic matrix determines the stabilizability of the network. The smaller the second largest eigenvalue is, the more the network is pinning controllable. Note that although the stability result obtained is given for a diagonalizable weighted network, it can be extended also to the case of non-diagonalizable networks which embeds at least an oriented spanning tree. Two representative examples are simulated, using non-chaotic systems and chaotic systems, respectively, as the nodes of the dynamical network with non-uniform coupling strengths, which demonstrate the effectiveness of the proposed V-stability criterion and stabilization scheme.

Acknowledgments

The authors are grateful to the anonymous reviewers for their valuable suggestions and comments, which have led to the improved presentation of this paper. This work was supported by the CNSF grant nos 60774088 and 60574036, the Specialized Research Fund for the Doctoral Program of Higher Education of China grant no 20050055013, the Program for New Century Excellent Talents of China (NCET-05–229), the Science and Technology Research Key Project of Education Ministy of China (no 107024).

References

[1] Watts D J and Strogatz S H 1998 Collective dynamics of 'small world' networks *Nature* **393** 440–2
[2] Barabási A-L and Albert R 1999 Emergence of scaling in random networks *Science* **289** 509–12
[3] Li C G and Maini P K 2005 An evolving network model with community structure *J. Phys. A: Math. Gen.* **38** 9741–9
[4] Li X and Chen G 2003 A local-world evolving network model *Physica* A **328** 274–86
[5] Chen G, Fan Z P and Li X 2004 *Modelling the complex Internet topology. Complex Dynamics in Communication Network* (Berlin: Springer)
[6] Chen F, Chen Z Q, Liu Z X, Xiang L Y and Yuan Z Z 2007 Finding and evaluating the hierarchical structure in complex networks *J. Phys. A: Math. Theor.* **40** 5013–23
[7] Wang X F and Chen G 2002 Synchronization in scale-free dynamical networks: Robustness and fragility *IEEE Trans. Circuits Syst. I* **49** 54–62
[8] Wang X F and Chen G 2002 Synchronization in small-world dynamical networks *Int. J. Bifurcation Chaos* **12** 1871–92
[9] Chavez M, Hwang D-U, Amann A and Boccaletti S 2006 Synchronizing weighted complex networks *Chaos* **16** 015106
[10] Motter A E, Zhou C and Kurths J 2005 Network synchronization, diffusion, and the paradox of heterogeneity *Phys. Rev.* E **71** 016116
[11] Lü J, Yu X and Chen G 2004 Chaos synchronization of general complex dynamical networks *Physica* A **334** 281–302
[12] Lü J and Chen G 2005 A time-varying complex dynamical network model and its controlled synchronization criteria *IEEE Trans. Autom. Control* **50** 841–6
[13] Lü J, Yu X, Chen G and Cheng D 2004 Characterizing the synchronizability of small-world dynamical networks *IEEE Trans. Circuits Syst.* I **51** 787–96
[14] Zhou J, Lu J and Lü J 2006 Adaptive synchronization of an uncertain complex dynamical network *IEEE Trans. Autom. Control* **51** 652–6
[15] Li C G and Chen G 2004 Synchronization in general complex dynamical networks with coupling delays *Physica* A **343** 263–78
[16] Zhou J and Chen T P 2006 Synchronization in general complex delayed dynamical networks *IEEE Trans. Circuits Syst.* I **53** 733–44
[17] Li Z and Chen G 2006 Global synchronization and asymptotic stability of complex dynamical networks *IEEE Trans. Circuits Syst. II* **53** 28–33
[18] Li Z and Chen G 2004 Robust adaptive synchronization of uncertain dynamical networks *Phys. Lett.* A **324** 166–78
[19] Li C G and Chen G 2004 Phase synchronization in small-world networks chaotic oscillators *Physica* A **341** 73–9
[20] Li X and Chen G 2003 Synchronization and desynchronization of complex dynamical networks: an engineering viewpoint *IEEE Trans. Circuits Syst. I* **50** 1381–90
[21] Comellas F and Gago S 2007 Synchronizability of complex networks *J. Phys. A: Math. Theor.* **40** 4483–92
[22] Yang J Z and Zhang M 2005 Chaos synchronization in complex networks *Chin. Phys. Lett.* **22** 2183–5
[23] Liu B, Liu X Z, Chen G and Wang H 2005 Robust impulsive synchronization of uncertain dynamical networks *IEEE Trans. Circuits Syst. I* **52** 1431–41
[24] Yang Z Q, Liu Z X, Chen Z Q and Yuan Z Z 2007 Adaptive synchronization of an uncertain complex delayed dynamical networks *Int. J. Nonlinear Sci.* **3** 93–100
[25] Sorrentino F, Bernardo M di, Cuéllar G H and Boccaletti S 2006 Synchronization in weighted scale-free networks with degree-degree correlation *Physica* D **224** 123–9

[26] Sorrentino F, Bernardo M di and Garofalo F 2007 Synchronizability and synchronization dynamics of weighted and unweighted scale free networks with degree mixing *Int. J. Bifur. Chaos* **17** 2419–34
[27] Hwang D-U, Chavez M, Amann A and Boccaletti S 2005 Synchronization in complex networks with age ordering *Phys. Rev. Lett.* **94** 138701
[28] Nishikawa T, Motter A E, Lai Y-C and Hoppensteadt F C 2003 Heterogeneity in oscillator networks: are smaller worlds easier to synchronize? *Phys. Rev. Lett.* **91** 014101
[29] Wu C W 2005 Synchronization in arrays of coupled nonlinear systems with delay and nonreciprocal time-varying coupling *IEEE Trans. Circuits Syst. II* **52** 282–6
[30] Barahona M and Pecora L M 2002 Synchronization in small-world systems *Phys. Rev. Lett.* **89** 054101
[31] Nishikawa T and Motter A E 2006 Synchronization is optimal in nondiagonalizable networks *Phys. Rev.* E **73** 065106
[32] Nishikawa T and Motter A E 2006 Maximum performance at minimum cost in network synchronization *Physica D* **224** 77–89
[33] Grigoriev R O, Cross M C and Schuster H G 1997 Pinning control of spatiotemporal chaos *Phys. Rev. Lett.* **79** 2795–8
[34] Sorrentino F, Bernardo M di, Garofalo F and Chen G 2007 Controllability of complex networks via pinning *Phys. Rev.* E **75** 046103
[35] Sorrentino F 2007 Effects of the network structural properties on its controllability *Chaos* **17** 033101
[36] Wang X F and Chen G 2002 Pinning control of scale-free dynamical networks *Physica* A **310** 521–31
[37] Li X, Wang X F and Chen G 2004 Pinning a complex dynamical network to its equilibrium *IEEE Trans. Circuits Syst. I* **51** 2074–87
[38] Liu Z X, Chen Z Q and Yuan Z Z 2007 Pinning control of weighted general complex dynamical networks with time delay *Physica* A **375** 345–54
[39] Xiang L Y, Liu Z X, Chen Z Q, Chen F and Yuan Z Z 2007 Pinning control of complex dynamical networks with general topology *Physica A* **379** 298–306
[40] Chen T, Liu X and Lu W 2007 Pinning complex networks by a single controller *IEEE Trans. Circuits Syst. I* **54** 1317–26
[41] Xiang J and Chen G 2007 On the V-stability of complex dynamical networks *Automatica* **43** 1049–57
[42] Pastor-Satorras R and Vespignani A 2001 Epidemic spreading in scale-free networks *Phys. Rev. Lett.* **86** 3200–3
[43] May R M and Lloyd A L 2001 Infection dynamics on scale-free networks *Phys. Rev.* E **64** 066112
[44] Polis G A 1998 Ecology: Stability is woven by complex webs *Nature* **395** 744–5
[45] Krause A E, Frank K A, Masom D M, Ulanowicz R E and Taylor W W 2003 Compartments revealed in food-web structure *Nature* **426** 282–5
[46] Latora V and Marchiori M 2001 Efficient behavior of small-world networks *Phys. Rev. Lett.* **87** 198701
[47] Latora V and Marchiori M 2003 Economic small-world behavior in weighted networks *Eur. Phys. J.* B **32** 249–63
[48] Pastor-Satorras R and Vespignani A 2004 *Evolution and Structure of the Internet: A Statistical Physics Approach* (Cambridge: Cambridge University Press)
[49] Guimerà R and Amaral L A N 2004 Modeling the world-wide airport network *Eur. Phys. J.* B **38** 381–5
[50] Li W and Cai X 2004 Statistical analysis of airport network of China *Phys. Rev.* E **69** 046106
[51] Yook S H, Jeong H and Barabási A-L 2001 Weighted evolving networks *Phys. Rev. Lett.* **86** 5835–8
[52] Motter A E, Zhou C and Kurths J 2005 Enhancing complex-network synchronization *Europhys. Lett.* **69** 334
[53] Lorenz E N 1963 Deterministic nonperiodic flow *J. Atmos. Sci.* **20** 130–41
[54] Chen G and Ueta T 1999 Yet another chaotic attractor *Int. J. Bifurcation Chaos* **9** 1465–6